Research Concerning the Use of Polynomial Functions in the Study of the Conventional Sucker Rod Pumping Units

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The analysis of the conventional sucker rod pumping units and especially their dynamics is laborious and requires a large amount of calculations due to the constructive complexity of the component structural elements. Therefore, expressing in a simplified but sufficiently precise form the various cinematic and dynamic parameters that characterize the proper functioning of the pumping units is extremely useful in analyzing the possibilities of their functional optimization. In the paper are presented a series of results concerning the expressing of some cinematic and dynamic parameters using polynomial functions in the case of the mechanism of a C-640D-305-120 pumping unit. The simulations have been performed with a computer program developed by the author using Maple programming environment. Experimental records processing was performed with the program Total Well Management.

Keywords: sucker rod pumping unit, kinematics, dynamics, polynomial functions

The analysis of the sucker rod pumping installations in the perspective of their functional optimization requires an accurate determination of the parameters that characterize the operation of the pumping unit mechanism and those corresponding to the movement of the sucker rod column [1-3]. On the other hand, especially the analysis of their dynamics is extremely laborious and requires a large amount of calculations due to the constructive complexity of the component structural elements of the pumping unit mechanism and due to the complex loads that appear during the movement of the sucker rod column [4-8]. Therefore, expressing in a simplified but sufficiently precise form the various cinematic and dynamic parameters that characterize the proper functioning of the sucker rod pumping installations is extremely useful in analyzing the possibilities of their functional optimization.

In this paper are presented some results concerning the use of the polynomial functions for expressing some cinematic and dynamic parameters corresponding to the operation of the mechanism of a C-640D-305-120 pumping unit. Some significant results concerning the cinematic and dynamic analysis of the mechanisms that have strongly helped to the achievement of the research from this paper are presented in [9-14]. The simulations have been performed with a computer program developed by the author using Maple programming environment [15] and the experimental records processing was performed with the program *Total Well Management* [16].

Experimental part

It has been analyzed a well serviced by a C-640D-305-120 pumping unit manufactured by *Lufkin* [17]. The experimental records have been processed using the program *Total Well Management* [16]. In establishing the polynomial function corresponding to the variation on a cinematic cycle of the motor moment at the crankshaft of the analyzed pumping unit using the simulation program mentioned before was used the record concerning the variation of the force at the polished rod for the stroke 70 (fig. 1).

The simulation results have been compared with the experimental records obtained for the variation on a cinematic cycle of the motor moment at the crankshaft for the strokes 70, 80 and 85 (fig. 2-4).

Establishing the polynomial function corresponding to the variation of the motor moment at the crankshaft

The mechanism of a conventional pumping unit is presented in figure 5. For establishing the variation on a cinematic cycle of the motor moment M_m at the crankshaft

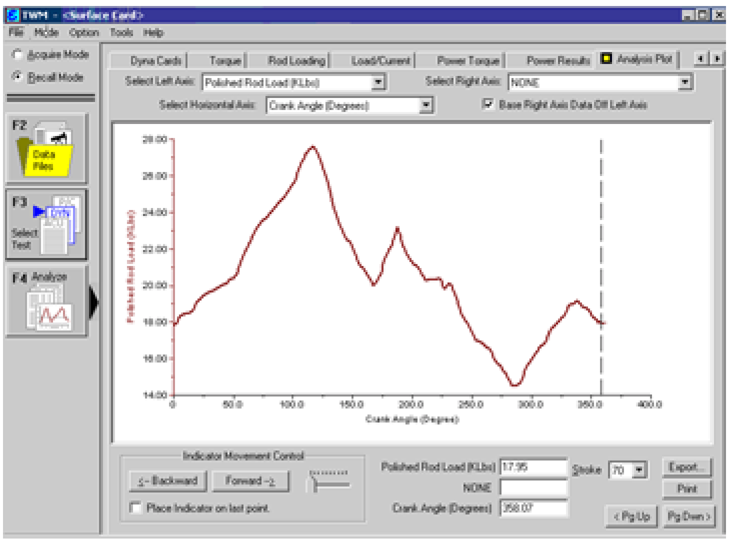

Fig. 1. The variation of the force at the polished rod during the stroke 70

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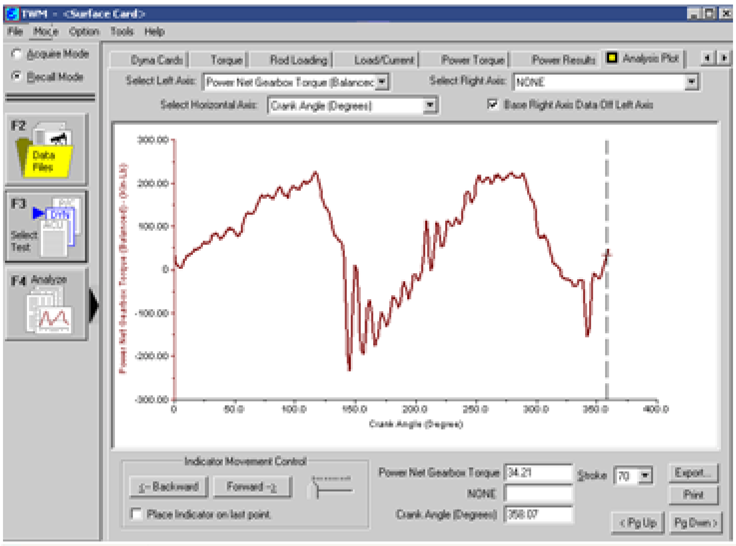

Fig. 2. The variation of the motor moment at the crankshaft during the stroke 70

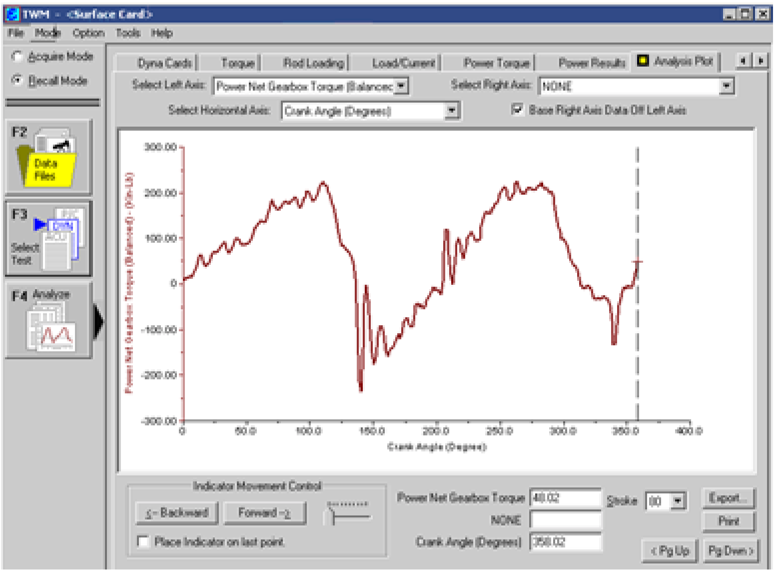

Fig. 3. The variation of the motor moment at the crankshaft during the stroke 80

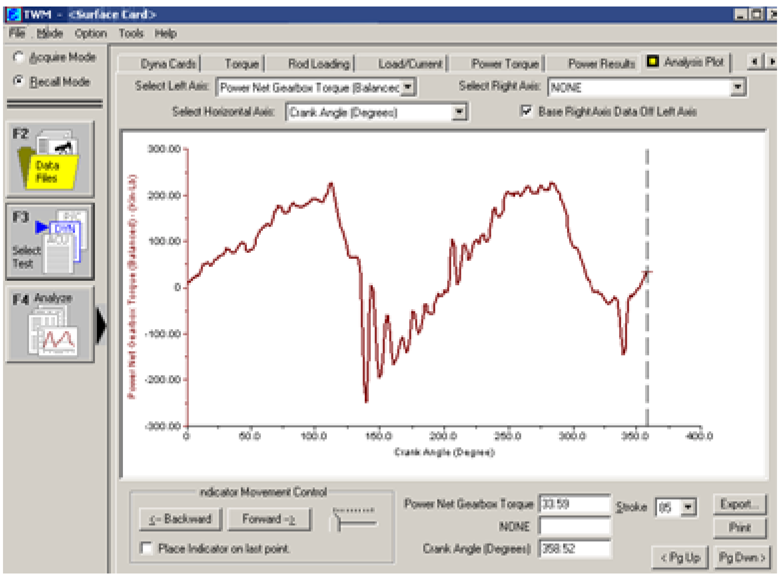

Fig. 4. The variation of the motor moment at the crankshaft during the stroke 85

are taken into account the weight G_1 of the cranks, the weight G_{CG} of the balancing counterweights and the force $\overline{F}$ acting at the end of the polished rod. In fig.ure5 with C_1 was noted the mass center of the cranks and m_{CG} represents the total mass of the balancing counterweights.

The motor moment M_m at the crankshaft is calculated by expressing the dynamic equilibrium in instantaneous powers with the following relation:

$$\overline{M}_m \cdot \overline{\omega}_1 + \overline{G}_1 \cdot \overline{v}_{C_1} + \overline{G}_{CG} \cdot \overline{v}_{A'} + \overline{F} \cdot \overline{v}_D = 0 \qquad (1)$$

where: ω_1 is the angular speed of the cranks; $\overline{v}_{C1} = \overline{\omega}_1 \times \overline{OC_1}$ is the speed of the mass center C_1 of the cranks; $\overline{v}_{A'} = \overline{\omega}_1 \times \overline{OA'_1}$ is the speed of the point where is acting the weight G_{CG} of the balancing counterweights; $\overline{v}_D$ is the speed of the end of the polished rod that can be established with the relation [6]: $v_{AD} = \omega_{3'}$. CD, where is the angular speed of the rocker that can be calculated with the following relation:

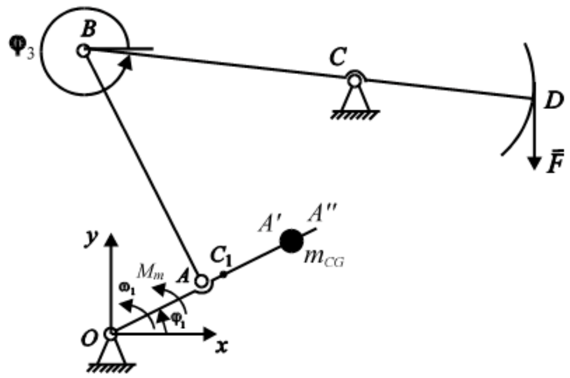

Fig. 5. The mechanism of a conventional pumping unit

$$\omega_3 = \dot{\varphi}_3 = \frac{d\varphi_3}{d\varphi_1} \cdot \frac{d\varphi_1}{dt} = \omega_1 \cdot \frac{d\varphi_3}{d\varphi_1} \qquad (2)$$

The manner of determining the variation of the angle φ_3 according to the angle φ_1 (fig. 5) is presented in [6].

The variation on a cinematic cycle of the force *F* at the end of the polished rod can be expressed according to the angle φ_1 with a polynomial function whose coefficients are calculated with the method of the least squares [18]:

$$F(\varphi_1) = c_0 + c_1 \cdot \varphi_1 + c_2 \cdot \varphi_1^2 + \ldots + c_m \cdot \varphi_1^m \qquad (3)$$

The values of the coefficients c_i, $i=\overline{0,m}$, that gives the best approximation of the variation on a cinematic cycle of the force *F* can be determined by solving the following system of equations [18]:

$$\begin{cases} c_0 \cdot (n+1) + c_1 \cdot \sum_{i=0}^{n} \varphi_{1i} + c_2 \cdot \sum_{i=0}^{n} \varphi_{1i}^2 + \ldots + c_m \cdot \sum_{i=0}^{n} \varphi_{1i}^m = \sum_{i=0}^{n} F(\varphi_{1i}) \\ c_0 \cdot \sum_{i=0}^{n} \varphi_{1i} + c_1 \cdot \sum_{i=0}^{n} \varphi_{1i}^2 + c_2 \cdot \sum_{i=0}^{n} \varphi_{1i}^3 + \ldots + c_m \cdot \sum_{i=0}^{n} \varphi_{1i}^{m+1} = \sum_{i=0}^{n} \varphi_{1i} \cdot F(\varphi_{1i}) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ c_0 \cdot \sum_{i=0}^{n} \varphi_{1i}^m + c_1 \cdot \sum_{i=0}^{n} \varphi_{1i}^{m+1} + c_2 \cdot \sum_{i=0}^{n} \varphi_{1i}^{m+2} + \ldots + c_m \cdot \sum_{i=0}^{n} \varphi_{1i}^{2m} = \sum_{i=0}^{n} \varphi_{1i}^m \cdot F(\varphi_{1i}) \end{cases} \qquad (4)$$

where φ_{1i} are the values of the angle φ_1 where have been recorded the values of the force at the polished rod.

In a similar way, $d\varphi_3 / d\varphi_1$ that appears in the calculus relation of the angular speed ω_3 and the trigonometric function $\cos\varphi_1$ that appears in the expressions of the terms $\overline{G_1} \cdot \overline{v_{C1}}$ and $\overline{G_{CG}} \cdot \overline{v_{A'}}$ in relation (1) can be expressed with polynomial functions, so that in the end is obtained the variation of the motor moment M_m expressed in a polynomial form.

Simulation results and discussions

The analysis of the mechanism of the conventional sucker rod pumping units presented before has been transposed by the author into a computer program using Maple programming environment [15]. The simulations have been performed in the case of a C-640D-305-120 pumping unit produced by *Lufkin* [17], whose component elements have the following dimensions: *OA* = 30 in. (0.762 m); *AB* = 133.5 in. (3.3909 m); *BC* = 111.09 in. (2.8217 m); *CD* = 155 in. (3.937 m). The coordinates of the point *C* (fig. 5) are [17]: x_c=111(2.8194 m) and y_c=138 in. (3.5052 m). The value of the crank angle φ_{1d} corresponding to the beginning of the upward movement of the sucker rod column is 88.976°[6].

The simulations have been accomplished by considering the following values of the other parameters involved in calculations: *OA'* = 46 in. (1.1684 m); *OA"* = 95 in. (2.413 m); m_{CG}=4808kg; q_1 = 722 kg/m, where q_1 is the linear mass of the cranks. The angular working speed of the cranks has been of 6.667 rot/min.

In evaluating the polynomial function corresponding to the variation of the force at the polished rod for the stroke 70 (fig. 1) it has been consider m=30. In figure 6 are presented the values of the coefficients c_i,$i=\overline{0,m}$, in the order in which they are found in the polynomial function.

[17.362060188105555322032169796623 2 , 51.596491284978833002682925621 1 , -1602.864286037982358847948537062 , 25835.748864512829853666076944 62 ,
-236662.788544538996620799568432 5 , 0.136660444775907293824141873394 6 10^{7} , -0.535622393766591132688418609453 10^{7} , 0.150330394680641034428170016 5
-0.314264022817558212684873127493 3 10^{8} , 0.503884994773069732803276199316 10^{8} , -0.633514871151885072665658680270 10^{8} , 0.635109601493705541644317
-0.514136145074300775811704871718 10^{8} , 0.339171292879536111640094905279 1 10^{8} , -0.183442241278090130055085601498 4 10^{8} , 0.815835185563913681193 04
-0.298180168460105925321514474525 10^{7} , 890895.282163225089395427822321 , -214802.139165159900897516670474 , 40633.836802533714980506622320 9 ,
-5629.285538397327002794949511 89 , 443.948728942439376104083603 97 , 21.162031833770473556652669640 , -13.867894164747495244565735772 5 ,
2.586159111286352376137532230 17 , -0.310318598522244108365948322808 , 0.026405321566078352212453223653 2 , -0.001601075416680539714930272884985 ,
0.000066341770250256149688822430596 1 , -0.169236183583102354115501219606 0 10^{-5} , 0.201124561075871755225017332245 8 10^{-7}]

Fig. 6. The values of the coefficients of the polynomial function corresponding to the force at the polished rod

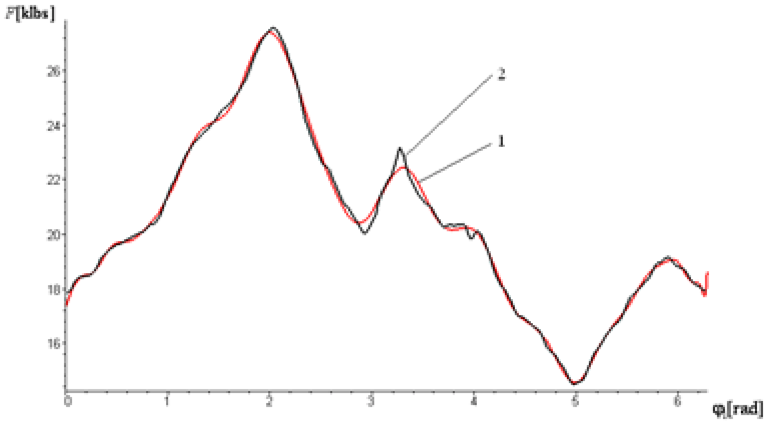

Fig. 7. The variation of the force at the polished rod during the stroke 70 (simulation -curve *1*; experimental - curve *2*)

2.3898139870053 -5.6105692327637 4.8381494974078 -2.00453474042993 0.45564888737925 -0.058984255139279 0.0040945832121619 -0.000118093698

Fig. 8. The values of the coefficients of the polynomial function corresponding to $d\varphi_3/d\varphi_1$

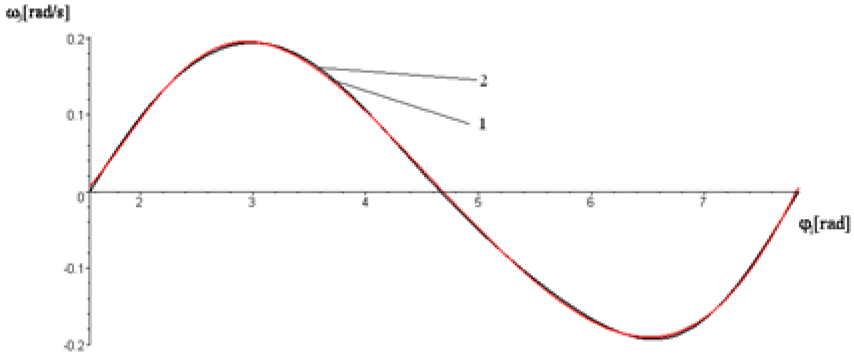

Fig. 9. The variation of the angular speed ω_3 during a cinematic cycle using the polynomial function (curve *1*) and the exact kinematics (curve *2*)

In figure 7 the curve *1* represents the variation of the force at the polished rod during a cinematic cycle beginning with the upward movement of the sucker rod column obtained using the polynomial function and the curve *2* corresponds to the variation of its measured values.

In evaluating the polynomial function corresponding to the variation of $d\varphi_3/d\varphi_1$ during a cinematic cycle beginning with the crank angle φ_{1d} it has been consider m=7. In figure 8 are presented the values of the coefficients c_i,i=0,m, in the order in which they are found in this polynomial function.

In figure 9 the curve *1* represents the variation of the angular speed during a cinematic cycle beginning with the crank angle φ_{1d} obtained using the polynomial function corresponding to $d\varphi_3/d\varphi_1$ and the curve *2* corresponds to the variation of its values calculated with relation (2).

In figure 10 are presented the values of the coefficients c_i,i=0,m, in the order in which they are found in the polynomial function that approximates during a cinematic cycle beginning with the crank angle φ_{1d} the trigonometric function $\cos\varphi_1$ that appears in the expressions of the terms $G_1.\overline{v}_{C1}$ and $G_{CG}.\overline{v}_{A'}$ in relation (1). In this case it has been consider m=6.

-1.39415489974741 4.36477424361387 -3.59346698547035 1.06195802915168 -0.127007215422111 0.00513608107449 34 0.000017387437409450

Fig. 10. The values of the coefficients of the polynomial function corresponding to $\cos\varphi_1$

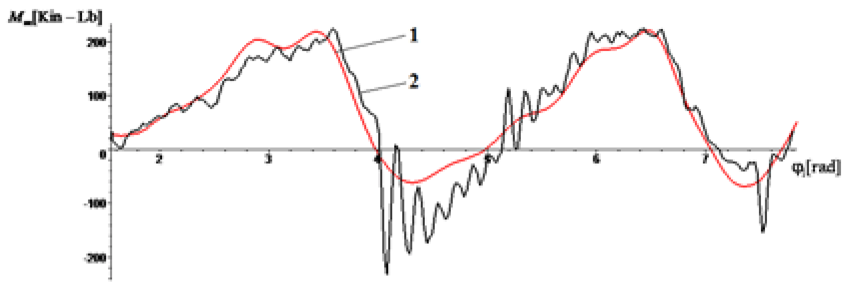

Fig. 11. The variation of the motor moment at the crankshaft during the stroke 70 (simulation -curve *1*; experimental - curve *2*)

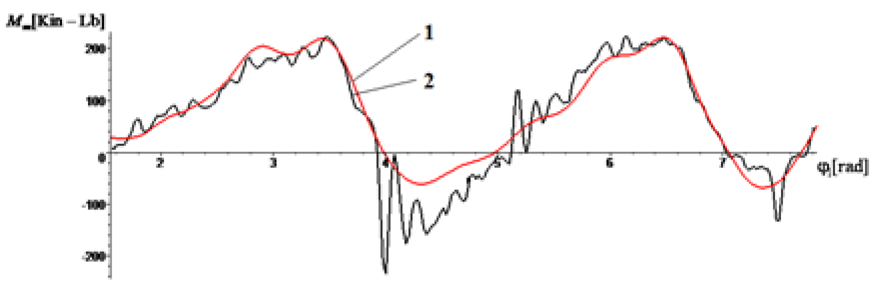

Fig. 12. The variation of the motor moment at the crankshaft during the stroke 80 (simulation -curve *1*; experimental - curve *2*)

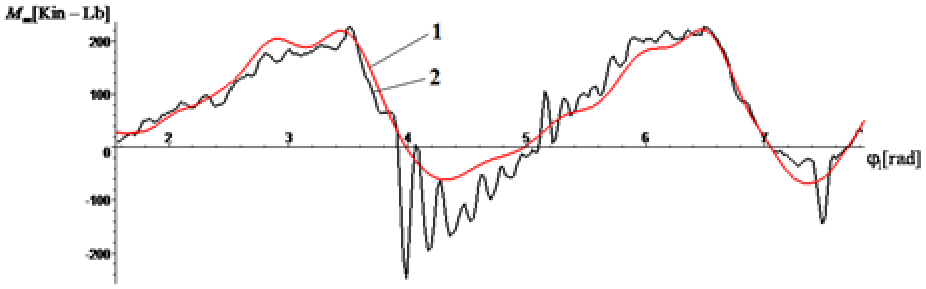

Fig. 13. The variation of the motor moment at the crankshaft during the stroke 85 (simulation -curve *1*; experimental - curve *2*)

In figures 11÷13 the curve *1* represents the variation during a cinematic cycle beginning with the crank angle φ_{1d} of the motor moment M_m at the crankshaft expressed in a polynomial form and the curves *2* corresponds to the variation of its measured values for the strokes 70, 80 and 85 respectively.

Figures 11÷13 highlight a good accordance between the values of the motor moment at the crankshaft obtained when it is expressed in the polynomial form and its measured values

Conclusions

In this paper were presented some results concerning the use of the polynomial functions in the operation analysis of the mechanism of the conventional pumping units. The program *Total Well Management* has been used for processing the experimental records and the simulations have been performed with a computer program developed by the author using Maple programming environment. After the simulations performed in the case of a C-640D-305-120 pumping unit it has been established the variation on a cinematic cycle of the motor moment at the crankshaft in a polynomial form after having previously obtained the polynomial functions corresponding to the variation on a cinematic cycle of the force at the polished rod and of the angular speed of the rocker. The simulation results highlighted a good accordance between the values of the motor moment at the crankshaft obtained when it is expressed in the polynomial form and its measured values.

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